Geometric Sequences

Look at the sequence: a = 2, 6, 10, 14, … What is the pattern? Add by 4 Type of Seq? arithmetic

Look at the sequence: a = 2, 8, 32, 128, … What is the pattern? Multiply by 4 Type of Seq? GEOMETRIC

A sequence where the ratio of consecutive terms is constant (IN OTHER WORDS, a sequence where you MULTIPLY or DIVIDE by the same number each time) is called a geometric sequence.

The number that you multiply by each time is called the common ratio.

EX 1: What are the next 3 terms and state the common ratio of the following geometric sequence.

a = 3, 6, 12, ____, ____, ____ » common ratio = $6 \div 3 = 2$ » a = 3, 6, 12, 24, 48, 96

EX 2: What are the next 3 terms and state the common ratio of the following geometric sequence.

a = 81, 27, 9, ___, ____, ____ » common ratio = $= 27 \div 81 = .333 = \frac{1}{3}$

(Note: ratio is in terms of multiplication) » a = 81, 27, 9, 3, 1, 1/3

Explicit form of a Geometric Sequence

a = 3, 6, 12, 24, …

$a_1 = 3 = 3 \times 2^0 =$ $3 \times 2^{(1-1)}$

$a_2 = 6 = 3 \times 2 = 3 \times 2^1 =$ $3 \times 2^{(2-1)}$

$a_3 = 12 = 6 \times 2 = 3 \times 2 \times 2 = 3 \times 2^2 =$ $3 \times 2^{(3-1)}$

$a_4 = 24 = 12 \times 2 = 3 \times 2 \times 2 \times 2 = 3 \times 2^3 =$ $3 \times 2^{(4-1)}$

$a_n = 3 \times 2^{(n-1)}$

The explicit formula for the geometric sequence is: $a_n = a_1 \times r^{(n-1)}$

Where $a_{1=}$ initial term and r = common ratio

Ex 3: Write the explicit formulas for the given sequences.

a = 5, 15, 45, … » $a_1 = 5,\ r = 3$ » $a_n = 5 \times 3^{(n-1)}$

$a = 25, -5, 1, -\frac{1}{5}, \ldots$ » $a_1 = 25,\ r = -\frac{1}{5}$ » $a_n = 25 \times \left(-\frac{1}{5}\right)^{(n-1)}$

Ex 4: Find the given term of the sequence.

$a_1 = 3$, r = 6, n = 11 » a_n =? » $a_n = 3 \times 6^{(11-1)} = 3 \times 6^{10} = 3 \times 60466176 = 181398528$

Ex 5: The given number is which term of the sequence?

128; $a_n = 0.5 \cdot (2)^{n-1}$ » n = ? » $128 = 0.5 \cdot (2)^{n-1}$ » $256 = 2^{n-1}$ (divide each side by 0.5) »

Convert to Logarithms: $n - 1 = \log_2 256$

$\log_2 256 = \frac{\log 256}{\log 2} = \frac{2.408}{0.301} = 8$ » so: n – 1 = 8 (add 1 to both sides) »

∴ n = 9